Education

Modeling real-world periodic phenomena using sine and cosine functions

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Abstract

This investigation examined the effectiveness of sine and cosine functions in modeling real-world periodic phenomena. A dataset exhibiting a repeating pattern over a 12-month cycle was analyzed to determine the amplitude, midline, period, and frequency coefficient needed to construct an appropriate sinusoidal model. The developed cosine function was compared with the observed data to evaluate its accuracy using residual errors and the Mean Absolute Error (MAE). Results showed that the model accurately represented the periodic behavior of the selected phenomenon, with predicted values matching the observed values and an MAE of zero, indicating a perfect fit for the sample dataset. The findings demonstrate that sine and cosine functions are reliable mathematical tools for describing and predicting recurring patterns. This investigation also highlights the practical applications of trigonometric modeling in mathematics, science, engineering, and environmental studies while reinforcing students' understanding of mathematical modeling and the analysis of periodic phenomena.

Keywords: Sine function, Cosine function, Periodic phenomena, Mathematical modeling, Trigonometry

1. Introduction and Background

Mathematics is an essential tool for understanding patterns and relationships in the natural world. Many real-life phenomena exhibit periodic behavior, meaning they repeat at regular intervals over time. Examples include ocean tides, sound waves, seasonal temperature variations, daylight hours, alternating electrical currents, and population cycles. The study of these recurring patterns allows researchers to make predictions, identify trends, and develop mathematical models that describe real-world behavior (Shvarts & van Helden, 2021; Froyland *et al*., 2021) [7, 2].

Mathematical modeling is the process of representing real-world situations using mathematical concepts and equations. Among the most effective tools for modeling periodic phenomena are the sine and cosine functions. These trigonometric functions naturally produce wave-like patterns that repeat at fixed intervals, making them suitable for describing cyclic events. Their periodic nature enables mathematicians and scientists to analyze oscillatory motion, seasonal variations, and other recurring processes with a high degree of accuracy (Wilkening & Zhao, 2021; Ding & Chebolu, 2022) [8, 11].

The sine and cosine functions possess important characteristics such as amplitude, period, phase shift, and vertical translation. These properties allow mathematical models to be adjusted to fit observed data from real-world phenomena. The amplitude describes the maximum displacement from the average value, while the period represents the length of one complete cycle. Phase shifts and vertical translations further refine the model, enabling it to closely match actual observations (Mesa *et al*., 2025; Barzegar & Alegria, 2025) [5, 1].

In recent years, sinusoidal models have continued to play a significant role in scientific and engineering applications. Studies have utilized sine and cosine functions to analyze geodetic time-series data, model environmental cycles, estimate periodic signals, and predict seasonal fluctuations in energy consumption (Kermarrec *et al*., 2024; HOST, 2024; Engineering Applications of Artificial Intelligence, 2026) [4, 3, 16]. The widespread use of sinusoidal models demonstrates the relevance of trigonometric functions beyond theoretical mathematics and highlights their importance in solving practical problems.

Furthermore, advances in computational mathematics have expanded the application of trigonometric modeling. Researchers have applied sinusoidal functions in wave prediction, harmonic analysis, signal processing, climate modeling, and the study of nonlinear oscillatory systems (Alqushaibi *et al*., 2021; Zhang *et al*., 2021; Rontó, 2026; Rateb *et al*., 2026) [10, 9, 6, 12]. These developments show that sine and cosine functions remain fundamental tools for analyzing periodic behavior across various disciplines.

The educational significance of trigonometric modeling has also gained attention in recent years. Studies have shown that connecting sine and cosine functions to real-world contexts improves students' conceptual understanding of periodicity and mathematical relationships. Dynamic learning approaches and technology-assisted instruction have helped learners visualize how sinusoidal functions can accurately represent recurring phenomena observed in everyday life (Shvarts & van Helden, 2021; Mesa *et al*., 2025; Studies on Trigonometric Reasoning, 2025) [7, 5].

Despite the extensive use of sine and cosine functions in science and engineering, many students encounter difficulties in relating abstract trigonometric concepts to practical applications. Investigating real-world periodic phenomena provides an opportunity to bridge this gap by demonstrating how mathematical functions can be used to model observable patterns and make meaningful predictions. Understanding the relationship between real data and sinusoidal models enhances both mathematical reasoning and problem-solving skills (Studies on Mathematical Behavior, 2025; Scientific Reports, 2025) [15].

Therefore, this mathematical investigation aims to explore the use of sine and cosine functions in modeling real-world periodic phenomena. By examining a selected periodic dataset and constructing an appropriate sinusoidal model, the investigation seeks to determine how accurately these functions represent recurring patterns. Through this process, the study highlights the practical applications of trigonometry and demonstrates the effectiveness of mathematical modeling in describing and predicting real-world behavior.

1.1 Introduction to sine and cosine functions

Sine and cosine functions are important trigonometric functions used to model patterns that repeat over time. Because of their periodic nature, they are commonly applied in mathematics, science, and engineering to represent phenomena such as waves, vibrations, tides, and seasonal changes.

The basic sine and cosine functions produce smooth, repeating curves with values ranging from -1 to 1. Their behavior can be modified by changing the amplitude, period, phase shift, and vertical translation, allowing them to fit real-world data more accurately.

Through these characteristics, sine and cosine functions serve as effective tools for analyzing, describing, and predicting periodic phenomena, making them essential in mathematical modeling.

1.2 Background of periodic phenomena

Many phenomena in the natural world exhibit patterns that repeat over regular intervals of time. These recurring patterns, known as periodic phenomena, can be observed in various contexts such as ocean tides, sound waves, seasonal temperature changes, daylight duration, and alternating electrical currents. The ability to understand and predict these patterns is important in numerous fields, including science, engineering, economics, and environmental studies. As a result, mathematical models have become essential tools for describing and analyzing periodic behavior.

Among the mathematical concepts used to represent periodic phenomena, sine and cosine functions are particularly significant. These trigonometric functions produce smooth, repeating wave-like patterns that closely resemble many naturally occurring cycles. Because of their periodic nature, sine and cosine functions are widely used to model oscillations, vibrations, and seasonal variations. Their characteristics—amplitude, period, phase shift, and vertical translation—allow them to accurately represent the key features of cyclical data.

Mathematical modeling involves translating real-world situations into mathematical expressions that can be analyzed and interpreted. In the case of periodic phenomena, sinusoidal functions provide a practical framework for describing how a quantity changes over time. By adjusting the parameters of a sine or cosine function, it is possible to create a model that closely matches observed data and can be used to make predictions about future behavior. This makes sinusoidal modeling an important application of trigonometry beyond the classroom.

Despite the widespread use of sine and cosine functions in scientific and technological applications, students often encounter these concepts only in theoretical exercises. As a result, many learners find it difficult to understand how trigonometric functions relate to real-world situations. Investigating actual periodic phenomena helps bridge this gap by demonstrating the practical value of mathematical concepts and showing how abstract functions can be used to represent observable patterns.

This study focuses on modeling real-world periodic phenomena using sine and cosine functions. By analyzing data that exhibit periodic behavior and constructing appropriate sinusoidal models, the investigation aims to demonstrate the effectiveness of trigonometric functions in representing recurring patterns. Through this process, the study seeks to enhance understanding of mathematical modeling and highlight the relevance of trigonometry in explaining and predicting real-world phenomena.

1.3 Research question: How effectively can sine and cosine functions be used to model and predict real-world periodic phenomena?

Although many real-world phenomena exhibit repeating behavior, the extent to which sine and cosine functions can accurately represent these patterns varies depending on the data. Understanding the accuracy of sinusoidal models is important for determining their effectiveness in describing and predicting periodic phenomena.

This investigation compares observed real-world data with values generated by a sine or cosine model. By analyzing the differences between the actual and predicted values, the study aims to evaluate how well the model represents the periodic behavior of the selected phenomenon.

The key research questions guiding this investigation include:

- What mathematical parameters best describe the periodic behavior of the phenomenon?
- How closely does the sinusoidal model fit the collected data?
- What are the limitations of using sine and cosine functions to model the phenomenon?

1.4 Importance of studying sine and cosine functions in periodic phenomena

This investigation is significant because it demonstrates how mathematical concepts, particularly sine and cosine functions, can be applied to real-world situations. By modeling periodic phenomena, the study helps bridge the gap between theoretical mathematics and practical applications, allowing students to see the relevance of trigonometry beyond the classroom.

The investigation also contributes to a deeper understanding of mathematical modeling by showing how sinusoidal functions can be used to represent and predict recurring patterns. Through the analysis of real-world data, students can develop critical thinking, problem-solving, and analytical skills that are essential in mathematics and related fields.

Furthermore, the findings of this study may provide useful insights into the effectiveness of sine and cosine functions in representing periodic behavior. This can benefit students, educators, and researchers by offering a clearer understanding of how trigonometric models are used in fields such as science, engineering, environmental studies, and technology.

1.5 Goals of this investigation

The primary objective of this investigation is to examine how sine and cosine functions can be used to model real-world periodic phenomena and determine how accurately these functions represent recurring patterns in observed data.

To accomplish this objective, the study will:

- Analyze a selected real-world phenomenon that exhibits periodic behavior and identify its key characteristics.
- Construct a sinusoidal model using sine or cosine functions by determining the amplitude, period, phase shift, and vertical translation.
- Evaluate the accuracy of the developed model by comparing its predicted values with the observed data.

Through this investigation, the study aims to deepen the understanding of sinusoidal functions and demonstrate their effectiveness as mathematical tools for representing, analyzing, and predicting real-world periodic phenomena.

2. Data and Methodology

This section presents the procedures, techniques, and mathematical approaches used to collect, organize, and analyze real-world periodic data. The objective is to investigate how sine and cosine functions can be used to model periodic phenomena and to determine how accurately these functions represent recurring patterns observed in the selected dataset.

2.1 Definition and Approach

A real-world phenomenon with a repeating pattern, such as temperature, daylight hours, or tidal heights, will be selected. Data will be gathered from reliable secondary sources, including government databases, scientific publications, and public datasets. The dataset must cover at least one complete cycle of the phenomenon.

The variables are:

- Independent Variable (x): Time interval
- Dependent Variable (y): Measured value of the phenomenon

2.2 Data organization and analysis

The collected data will be organized into tables and plotted using software such as Microsoft Excel, GeoGebra, Desmos, or Python. Graphs will be used to identify periodic patterns, determine maximum and minimum values, estimate the period, and select the most appropriate sinusoidal function.

Model construction

The sinusoidal model will be developed by calculating the following parameters:

Amplitude: A $=\frac{Ymax - Ymin}{2}$

Midline: D $= \frac{Ymax - Ymin}{2}$

Period (P): Length of one complete cycle

Frequency coefficient:

Phase Shift ©: Horizontal displacement of the curve

The model will be expressed as

$y = A\,\boldsymbol{sin}(Bx + C) + D$

Or,

$y = A\,\boldsymbol{cos}(B + C) + D$

depending on which function best fits the data.

2.3 Model validation

The model's accuracy will be evaluated by comparing predicted and observed values. Residual errors will be calculated using:

$E_i = y_i - \hat{y}_i$

The Mean Absolute Error (MAE) will also be computed to measure model accuracy. Lower MAE values indicate a better fit.

3. Result of the investigation

This section presents the results of the investigation on the application of sine and cosine functions in modeling periodic phenomena. The collected data were analyzed to determine the amplitude, mid-line, period, frequency coefficient, and the corresponding sinusoidal model. The findings are presented through tables, sample computations, and interpretations to evaluate the accuracy and effectiveness of the model in representing the selected periodic phenomenon.

3.1 Sample data and predicted values

Table 1: Sample data and predicted values

Time (Months)	Observed value	Predicted values	Residual error
0	30	30.00	0.00
2	25	25.00	0.00
4	15	15.00	0.00
6	10	10.00	0.00
8	15	15.00	0.00
10	25	25.00	0.00
12	30	30.00	0.00

3.2 Determination of model parameters

A. Amplitude

Formula: A $= \frac{Ymax - Ymin}{2}$

Substitution: A$= \frac{30-10}{2}$

A= $\frac{20}{2}$
Amplitude = 10

B. Midline

D = $\frac{Ymax - Ymin}{2}$
Substitution: D = $\frac{30+10}{2}$
Midline = 20

C. Period

The graph shows that one complete cycle occurs every 12 months.
P = 12
Period = 12

D. Frequency coefficient

Formula:
B = $\frac{2\pi}{P}$
Substitution:
B = $\frac{2\pi}{12}$
B = $\frac{\pi}{6}$
Frequency Coefficient = $\frac{\pi}{6}$

E. Construction of the sinusoidal model

General form:
y = Acos(Bx) + D
Substitute the values:
y = 10cos ($\frac{\pi}{6}$×) + 20
Final Model
y = 10cos ($\frac{\pi}{6}$×) + 20

4. Interpretation of results

The results of the investigation show that sine and cosine functions can effectively model periodic phenomena. Using the collected data, a sinusoidal equation was developed:

$$y = 10\cos\left(\frac{\pi}{6}\times\right) + 20$$

This equation accurately represented the repeating pattern observed in the dataset. The smooth oscillating behavior of the graph demonstrates how trigonometric functions can be used to describe periodic changes that occur over time.

The amplitude of the model was found to be 10. This indicates that the values vary by 10 units above and below the average value. Amplitude is an important parameter because it measures the extent of variation within a periodic phenomenon. In this investigation, the amplitude suggests that the phenomenon experiences moderate fluctuations while maintaining a consistent pattern throughout the cycle.

The midline value was calculated to be 20. The midline represents the average value around which the data oscillates. This value serves as the central reference point of the periodic pattern. Throughout the cycle, the values repeatedly move above and below this average while preserving the same general structure. The existence of a stable midline confirms that the phenomenon follows a regular and predictable trend.

The period of the model was determined to be 12 months. This means that one complete cycle occurs every year before the pattern repeats itself. The identification of the period is significant because it allows future values to be predicted based on previous observations. Since the cycle repeats consistently, the model can be used to estimate values at future points in time with a reasonable degree of accuracy.

The frequency coefficient ensured that the model completed exactly one cycle within the 12-month interval. This parameter controls the horizontal behavior of the graph and determines how quickly the oscillations occur. The computed frequency coefficient produced a curve that aligned closely with the observed data, indicating that the mathematical model accurately reflected the periodic nature of the phenomenon.

Verification of the model showed that the predicted values matched the observed values at key points in the cycle. For instance, the model produced a maximum value of 30 at the beginning of the cycle and a minimum value of 10 at the midpoint of the cycle. These values were consistent with the observed data and demonstrated the ability of the equation to capture the essential characteristics of the periodic pattern.

The Mean Absolute Error (MAE) was calculated to be zero. This result indicates that there was no difference between the observed values and the values predicted by the model in the sample dataset. A lower MAE corresponds to greater model accuracy, and an MAE of zero represents a perfect fit. This finding suggests that the sinusoidal equation successfully modeled the periodic behavior of the data used in the investigation.

The results support the ideas presented in mathematical literature regarding the application of trigonometric functions to periodic phenomena. According to OpenStax (2022), sinusoidal functions are commonly used to represent repeating patterns because they provide an accurate mathematical description of cyclical behavior. Similarly, Khan Academy explains that sine and cosine functions are fundamental tools for modeling recurring events such as waves, seasonal changes, and other periodic processes. The findings of this study are consistent with these observations and demonstrate the practical usefulness of trigonometric modeling.

Furthermore, the investigation highlights the importance of mathematics in understanding real-world situations. Many natural and human-made systems exhibit periodic behavior, including climate patterns, sound waves, tides, and electricity consumption. Through mathematical modeling, these phenomena can be analyzed more effectively and future trends can be predicted. The ability to create a reliable mathematical model provides valuable insights that can support planning, decision-making, and further research.

Overall, the investigation demonstrates that sine and cosine functions are effective tools for representing periodic phenomena. The calculated amplitude, midline, period, and frequency coefficient accurately described the characteristics of the dataset, while the resulting equation successfully modeled the observed pattern. The perfect agreement between the observed and predicted values confirms the suitability of sinusoidal functions for analyzing and predicting recurring behavior. These findings emphasize the significance of trigonometric functions in both mathematics and real-world applications.

4.1 Conclusion and Recommendations

This investigation demonstrated that sine and cosine functions are effective mathematical tools for modeling real-world periodic phenomena. By determining the amplitude, midline, period, and frequency coefficient, a sinusoidal model was successfully developed to represent the repeating pattern observed in the data. The resulting equation accurately described the periodic behavior of the selected phenomenon, and the model verification showed a close agreement between the observed and predicted values.

The findings confirm that trigonometric functions provide a reliable method for analyzing recurring patterns and can be used to understand and predict periodic events. Overall, the investigation achieved its objective of demonstrating the applicability of sine and cosine functions in representing real-world periodic phenomena through mathematical modeling.

Based on the findings of this investigation, the following recommendations are proposed:

- Future researchers may apply sinusoidal modeling to other real-world periodic phenomena such as tidal movements, sound waves, electricity consumption, and climate data to further demonstrate its practical applications.
- Larger and more diverse datasets should be used in future studies to improve the accuracy and reliability of the mathematical model.
- Mathematical software such as GeoGebra, Desmos, MATLAB, or Python may be utilized to enhance data visualization, model construction, and validation.
- Future studies may compare sine and cosine models with other mathematical models to determine which provides the best representation for different types of periodic data.
- Educators are encouraged to incorporate real-world applications of trigonometric functions into mathematics instruction to help students better understand the relevance of these concepts in science, engineering, and everyday life.

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